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Abstract

Combinatorial auctions are important market mechanisms aimed at the efficient trade of multiple heterogeneous items. In this paper, we establish that in the design of combinatorial auctions, several complex issues, which go beyond the complexity of solving the allocation problem, may be encountered. These issues are mainly related to the formulation and the classification of combinatorial auctions in different settings, the expression of combined bids, and to the "tâtonnement" processes. For each category of issues, we survey the existing literature and propose some research avenues.

Keywords : combinatorial auctions, market design, bidding vocabulary, tâtonnement processes

Résumé

Les enchères combinatoires constituent une classe importante de mécanismes de marché visant les négociations de biens ou de services multiples et hétérogènes. Nous soutenons que les défis associés à la spécification et le déploiement d'enchères combinatoires vont bien au-delà du problème de la détermination du vainqueur. Nous examinons les formulations et une classification des enchères combinatoires, la définition de vocabulaires d'écriture de mises combinées, ainsi que la conception des mécanismes de tâtonnement. Pour chaque problème, nous faisons une brève revue de la littérature et nous identifions des avenues de recherche.

Mots-clés : enchères combinatoires, design de marché, vocabulaire d'écriture des mises combinées, mécanismes de tâtonnement

1 Introduction

Auctions are important market mechanisms, used from times immemorial for the allocation of goods and services. However, a significant increase in popularity has been observed during the last decade. This fact is mainly a result of the emergence of e-commerce and of a deregulation wave that led to the privatization of several industries (the FCC auction of telecommunication air waves in the US being the most striking example, Milgrom (1995, 1997)). Other important real-world applications of auctions include railroad right-of-way allocation (Brewer and Plott 1996), take-off and landing time slot allocation in airports (Rassenti, Smith, and Bulfin 1982), financial markets (Srinivasan, Stallaert, and Whinston 1998, and Fan, Stallaert, and Whinston 1999), and procurement of transportation services (Ledyard *et al.* 2000). The design of efficient auction mechanisms is acknowledged nowadays to be a critical step in the process of building electronic marketplaces.

In most markets of interest, several items of different nature are negotiated simultaneously. Often, a participant's valuation of a given item depends of whether it obtains other items as well. For instance, car manufacturers, which need several parts to build their car engines (engine-blocks, valves, bolts, etc.), are likely to valuate a bundle of parts making a complete engine more than its individual components taken separately.

The interdependency of items can take two basic forms: *complementarity* and *substitutability*. Let A and B be two items, and $v(\cdot)$ be the buyer's valuation function. A and B are said to be complementary if $v(\{A, B\}) > v(\{A\}) + v(\{B\})$, and substitutable if $v(\{A, B\}) < v(\{A\}) + v(\{B\})$. It is noteworthy that complementarity and substitutability may arise, in the same auction, for different trading situations. Consider auctions of time slots in airports as an example. A take-off slot associated to the origin airport of a flight and a corresponding landing slot in the destination airport indeed complement each other. On the other hand, two pairs of take-off and landing time slots, which correspond to the same origin and destination airports within the same period of time (e.g., from 8h00 to 8h30), are likely to be substitutable for an airline company interested in launching a daily service between the two airports.

Multi-round simultaneous ascending auctions are one of the most used mechanisms for trading multiple heterogeneous items. An auctioneer sells items to several buyers, which are permitted to submit simultaneous bids on the single items in which they are interested. At the end of each round, temporary winners and standing high bids for each item are announced, and the bidders are given the opportunity to revise their bids in order to beat the standing high bids. The auction ends when no new bids are submitted. Admissibility and activity rules often determine the limits within which buyers may define and modify their bids (see the analysis of Milgrom 1997, for example). Nevertheless, the fact the bidders are allowed to submit bids only on single items is a major limitation of the mechanism. In particular, a buyer may face an *exposure problem* in the presence of complementarity. In an unsuccessful attempt to secure a desirable bundle, the buyer may find itself stuck with a partial bundle paid more than its valuation of it. Buyers actually avoid that risk

by not expressing their true valuations for bundles, which leads to economically inefficient auctions. *Combinatorial auctions*, where buyers may bid on bundles of items, prevent this problem by allowing buyers to express exactly their preferences. On the other hand, they are computationally intractable in the general case: when n items are on sale, each buyer may submit up to $2^n - 1$ bids and winner determination is an NP-hard problem (Rothkopf, Pekeč, and Harstad 1998). Similar issues and challenges arrise in all other types of auctions (descending, multi-lateral, etc.).

The revenue maximization problem, in which the auctioneer determines an allocation of items that is the most profitable to itself, has received most of the attention in the literature on combinatorial auctions. Among the fundamental early works, Rothkopf, Pekeč, and Harstad (1998) propose a dynamic programming approach for a formulation of the revenue maximization problem as a set packing problem, which complexity is in $O(3^m)$, and suggest to restrict the set of combinations of items the buyers are allowed to bid on. The authors show that some special instances of the problem (problems with nested structures, as well as some cardinality-based and geometry-based structures) may be solved in polynomial times. A somewhat different approach due to Sandholm (1999) assumes no bid restrictions but rather capitalizes on the sparseness of the space of bids actually submitted and proposes a procedure to generate a tree representation of the set of feasible solutions and an efficient search algorithm. Other approaches include the AUSM mechanism introduced by Banks, Ledyard, and Porter (1989), the RAD auction of DeMartini *et al.* (1999), and the iBundle mechanism (Parkes 2000). AUSM is a combinatorial auction with asynchronous bid submission. It implements a greedy allocation mechanism: a bid is designated as a new winner if its price offer is higher than that of the other bids it conflicts with. The real novelty of AUSM is that buyers are allowed to form coalitions to displace winning bids. RAD is a combinatorial mechanism inspired from both AUSM and the Simultaneous Ascending Auction (Milgrom 1997). Finally, iBundle is an iterative ascending-price combinatorial auction which provably computes optimal allocations of items assuming best-response bidding strategies from buyers.

The work on Operations Research models and methods for determining allocations and prices in combinatorial auctions is thus in its infancy still. Moreover, we strongly believe the challenge of combinatorial auctions to be significantly more important and complex. In fact, while the winner determination in combinatorial auctions is an important problem, it is certainly not the only one. In this paper, we identify some of the other complex issues related to the design of efficient combinatorial auctions. The emphasis is on three issues: a classification of combinatorial auctions and the associated formulations; the expression of combinatorial bids; and the design of multi-round, tâtonnement mechanisms intended to determine allocations and prices in situations where complete information about the participants' valuations is not available.

The paper is organized as follows. In Section 2, we classify combinatorial auctions and present several formulations that show the diversity of real-world situations that may be encountered and designated as "combinatorial auctions". In Section 3, we tackle the important issue of the expression of combined bids and give evidence of the need for a bidding vocab-

ulary that goes beyond what the basic models currently permit participants to express. In Section 4, we discuss tâtonnement mechanisms that approximate the behavior of an "ideal", complete information market, when only incomplete information about participants' valuation functions is available. For these mechanisms, pricing and the design of auction rules are interesting but generally challenging issues that need to be studied more extensively. We conclude with a general discussion about the role of advisors to participants in combinatorial auctions.

2 Problem classification and formulations

Combinatorial auctions involve several participants that want to trade (sell, buy or both) multiple heterogeneous items, and an auctioneer that is a third party in charge of conducting the auction. The participants submit *combined bids* (all we need to know for the moment is that a combined bid is related to a collection of items rather than single items). The auctioneer, through an *auction* mechanism, must determine an *allocation of the items* to the participants as well as the corresponding *prices* to be paid.

It is possible to classify combinatorial auctions with respect to several criteria. From an Operations Research market design perspective, the most important are:

- The *nature* of the items traded:
  1. Indivisible goods versus infinitely divisible ones. For example, capacity in telecommunication networks is divisible, but rail right-of-way is not. While this distinction is mostly related to the physical nature of the goods, it is noteworthy that auctions of divisible goods can be appropriate even if the goods are physically indivisible, especially when large quantities of goods are traded (this is the case, for instance, with asset trading in financial markets).
  2. Pure commodities versus network ones. The latter usually designates situations where the services or capacity to be traded belong to systems with network structures.
  3. Procurement of services versus trade of regular goods.
- The *role of the participants* in the auction. It is possible to distinguish between *one-sided* auctions, in which there is only one seller or buyer (the auctioneer) and all the participants are buyers or sellers, and *double-sided, multilateral* auctions where each participant may sell and buy items, sometimes in the same bid.
- The *objectives* of the auction. Auctions can be *optimized* or not. Two main types of objectives are generally encountered when allocations and payments are determined in optimized auctions. In *revenue maximizing* auctions, the auctioneer is interested in the allocation that is the most profitable to itself. On the other hand, outcomes of *socially efficient* auctions are such that the overall utility of the participants is maximized.

- The *choreography* of the auction (Robert and Bérubé 2001) that determines if and how bids are submitted and eventually modified. Thus, for example, one distinguishes between *single-round, sealed-bid* auctions and *multi-round, open* auctions. In the former, participants submit sealed bids to the auctioneer only once and final allocations and prices are determined. Whereas in multi-round auctions, participants are allowed to modify their bids at the end of each round on the basis of information disclosed to them about the state of the auction. We further address these issues in Section 4.

Different models have generally to be derived for each of these categories. To illustrate the concepts and challenges, we present in the remainder of this section two main formulations and several variants that describe two extreme situations: a model for a one-sided combinatorial auction of indivisible goods and a formulation of a multilateral combinatorial auction of divisible goods.

2.1 The one-sided indivisible case

Let G be a set of m different items (available each in single units). Suppose one seller (the auctioneer) and a set N of n buyers involved in the auction.

We define a *bundle* S as a non-empty subset of items, $S \subseteq G, S \neq \emptyset$. Let $\delta_{i,S} = 1$ if item i belongs to S and 0 otherwise.

A buyer j has a valuation $v_j(S)$ for each bundle S. In order to obtain desirable items, buyers submit bids, that is price offers, on some or all bundles. Let $p_j(S)$ denote the bid of buyer j on $S \subseteq G, S \neq \emptyset$. It should be emphasized that bids $p_j(S)$ are not necessarily identical to valuations $v_j(S)$. We assume that $v_j(S) \geq 0$ and $p_j(S) \geq 0$, $\forall S \subseteq G, S \neq \emptyset$, $\forall j \in N$. We suppose also that the auction is run on a single-round, sealed-bid basis, so that the auctioneer collects all the bids and determines allocations and prices only once.

Let $x_{j,S} = 1$ if the bundle S is allocated to buyer j, and 0 otherwise. The two basic formulations of the optimized market clearing mechanism may then be written as:

The *revenue maximization* model (M1)

$$\max \quad \sum_{j \in N} \sum_{S \subseteq G; S \neq \emptyset} p_j(S) x_{j,S} \tag{1}$$

$$s.t. \quad \sum_{S \subseteq G; S \neq \emptyset} x_{j,S} \leq 1, \forall j \in N \tag{2}$$

$$\sum_{j \in N} \sum_{S \subseteq G; S \neq \emptyset} \delta_{i,S} x_{j,S} \leq 1, \forall i \in G \tag{3}$$

$$x_{j,S} \in \{0,1\}, \forall S \subseteq G, S \neq \emptyset, \forall j \in N \tag{4}$$

The *social efficiency maximization* model (M2)

$$\max \quad \sum_{j \in N} \sum_{S \subseteq G; S \neq \emptyset} v_j(S) x_{j,S} \tag{5}$$

$$quads.t. \quad \sum_{S \subseteq G; S \neq \emptyset} x_{j,S} \leq 1, \forall j \in N \tag{6}$$

$$\sum_{j \in N} \sum_{S \subseteq G; S \neq \emptyset} \delta_{i,S} x_{j,S} \leq 1, \forall i \in G \tag{7}$$

$$x_{j,S} \in \{0,1\} \quad S \subseteq G, S \neq \emptyset, \forall j \in N \tag{8}$$

Constraints (2) and (6) enforce the requirement that at most one bundle is allocated to any buyer. Relations (3) and (7) ensure that no item is allocated to more than one buyer. When buyers pay their bids for the bundles allocated to them, the objective of model $(M1)$ is to maximize the revenue of the auctioneer, while in model $(M2)$, the objective corresponds to the maximization of the overall utility the buyers derive from the bundles they obtain.

Notice that in both models, the auctioneer does not require all items to be allocated. Thus, in model $(M1)$ for example, if an item i is not part of any bundle S such that $p_j(S) > 0$ for a participant $j \in N$, then item i remains unsold. Also note that the non-negativity assumptions $p_j(S) > 0$ and $v_j(S) > 0$, $\forall j \in N, \forall S \subseteq G$ are required for the correctness of the constraint formulations. Indeed, when $p_j(S) < 0$, which means that participant j requires to be paid if it receives bundle S, as in some privatization-related auctions, some of the inequality constraints (3) and (7) have to be changed to equality, to reflect the auctioneer's requirement that the corresponding items have to be assigned. These modifications have no impact, however, on the issues addressed in the rest of the paper and are thus not discussed further.

2.2 Network formulations

In many cases, traded goods or services correspond to network resources. It is hoped that market allocations could be obtained more efficiently in such cases by using specialized network algorithms. We therefore present a formulation of a combinatorial auction of network capacity that, without being related to any particular application, provides insights into this type of markets.

Let $G = (V, A)$ be a network, where V is a set of vertices and A a set of links. To each link $a \in A$ is associated a capacity v_a. As in the pure item models considered above, we suppose that the capacity is owned by the auctioneer and that there are several buyers. The combinatorial aspect of the problem derives from the fact that buyers desire to obtain capacity between pairs of vertices, rather than on individual links. To simplify the presentation, we define a bid B_j submitted by buyer $j \in N$ as $B_j = (\{O_j, D_j\}, c_j, G_j, p_j)$ (we suppose, with no loss of generality, that a buyer submits a single bid), where

1. $(\{O_j, D_j\})$ is an origin-destination pair of vertices specifying that buyer j needs capacity between O_j and D_j;
2. c_j is the required capacity between O_j et D_j;
3. $G_j \subseteq G$ is a subnetwork such that $O_j, D_j \in G_j$, with the condition that all the capacity required between O_j and D_j must be within paths in G_j;
4. p_j is the price offer of participant j for the bundle.

Figure 1 illustrates such capacity bidding. Two bids have been submitted. B_1 is a \$100 bid for a capacity of 20 contained in the sub-network G_1 between O_1 and D_1, while B_2 is a \$80 bid for a capacity of 10 on path $O_2 - I_2 - D_2$.

Let K_j be the set of paths between O_j and D_j that are in G_j. We define the decision variables x_j, $\forall j \in N$ and h_k, $\forall k \in K_j$, $\forall j \in N$, as follows:

$$x_j = \begin{cases} 1, & \text{if the bid } B_j \text{ is winning,} \\ 0, & \text{otherwise;} \end{cases}$$

and h_k is the capacity allocated to participant j on path $k \in K_j$.

To shorten the presentation, we limit ourselves to the revenue maximization variant of

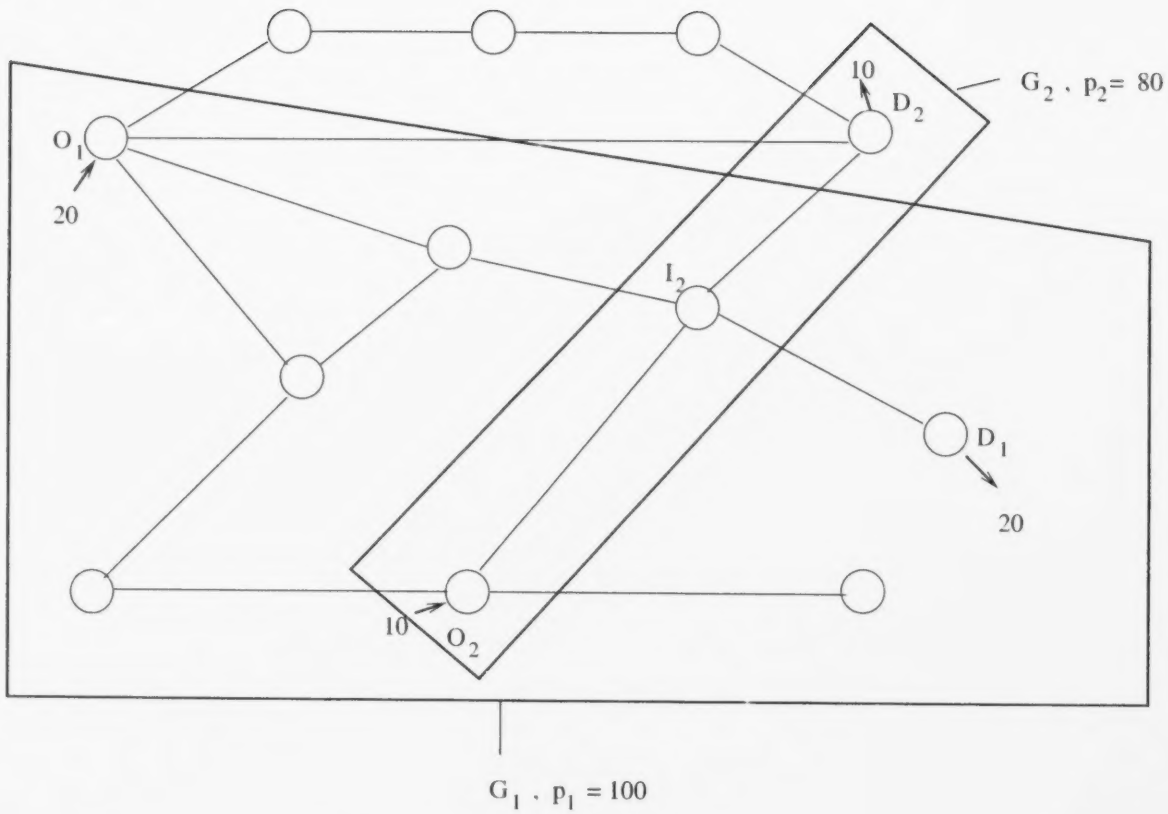

Figure 1: Combinatorial bids on capacity

capacity allocation. The model, $(M3)$, may be formulated as follows:

$$\max \quad \sum_{j \in N} x_j p_j \tag{9}$$

$$s.t. \quad \sum_{k \in K_j} h_k = c_j x_j, \forall j \in N \tag{10}$$

$$\sum_{j \in N} \sum_{k \in K_j} \delta_{ak} h_k \leq v_a, \forall a \in A \tag{11}$$

$$x_j \in \{0, 1\}, \forall k \in K_j, \forall j \in N \tag{12}$$

$$h_k \geq 0, \forall k \in K_j, \forall j \in N \tag{13}$$

where

$$\delta_{ak} = \begin{cases} 1, & \text{if link } a \in k, \\ 0, & \text{otherwise.} \end{cases}$$

Constraints (10) state that the capacity allocated to a winning bid must be within the bid's sub-network. Constraints (11) are capacity availability constraints.

2.3 The multilateral divisible case

Let G be a set of divisible items and N a set of participants to the auction. A bundle bid l submitted by a participant may take the form $B_l = (\{q_l^i, s_l^i\}_{i \in G}, p_l)$, where

- q_l^i is the maximum quantity of the item $i \in G$ that the participant is willing to trade in the bid l;
- s_l^i designates the nature of the trade with respect to bid l: $s_l^i = +1$ if the item i is bought and $s_l^i = -1$ if it is sold;
- p_l is a price offer for the bundle. Notice that $p_l > 0$ if the bid l is "globally" a bid to buy, while $p_l < 0$ if l is globally a bid to sell.

Implicit in this definition is the hypothesis that a participant may obtain fractions of its bids. For instance, a bid $B = (\{+10, -20, +5\}, -100)$ (for the sake of readability, the s_l^i have been assigned to the q_l^i) means that the trader is willing to sell *at most* 20 units of item 2, and to buy *at most* 10 units of item 1 and 15 units of item 3, and asks \$100 if the package is traded in its entirety. An allocation $\{+2, -4, +1\}$, which corresponds to a 20% execution of the bid, is assumed to be acceptable to the trader.

The auctioneer intends to "clear" the market, that is, to balance sell and buy orders, while reflecting the fact that priority in the execution of orders should be given to the most competitive orders, i.e., buy orders with highest price offers and sell orders with lowest ask

prices. Let L_j be the set of the bundle bids submitted by the participant $j \in N$ and define the decision variables

$$x_l : \text{the traded proportion of the bundle bid } l,\ l \in L_j,\ j \in N.$$

A market clearing model may then be formulated as follows

$$\max \quad \sum_{l \in L_j} \sum_{j \in N} x_l p_l \tag{14}$$

$$(M4) \quad s.t. \quad \sum_{l \in L_j} \sum_{j \in N} s_l^i q_l^i x_l = 0, \forall i \in G \tag{15}$$

$$x_l \in [0,1], \forall l \in L_j, \forall j \in N \tag{16}$$

Constraints (15) state that, for each item, buy orders in bundle bids have to be matched with sell orders. The objective of the model is to maximize the overall surplus of the market.

3 Bidding vocabularies

The framework that describes how bids are defined in a combinatorial market should be sufficiently powerful to allow the representation of the preferences and objectives of the various participants. From a market design perspective, it should be also flexible and general so that one does not have to invent a new vocabulary for every new application. In our opinion, the current state of bidding vocabularies does not adequately answers all these challenges, as we aim to illustrate in this section.

The definition of bidding vocabularies is also closely related to issues relative to the user interfaces and how easy it is for auction participants to enter their bids. The study of these issues is, however, outside the scope of the current paper.

3.1 Motivation

The definition of a combined bid as a bundle of items and a price offer in the basic models of the previous section is arguably too restrictive and far from being able to support the range of bidding requests that can be encountered in real situations.

In particular, "indifference" towards items or bundles of items, which is a special case of substitutability, is not captured very well. More explicitly, consider a combinatorial auction with two bidders, 1 and 2, and four items A, B, C, and D. Bidder 1 makes the following statement: "I want the bundle $\{A, B\}$ at \$50, or the bundle $\{C, D\}$ at \$30, *but not both of them*", while bidder 2 asks for $\{B\}$ at \$30. With no mean to express the exclusion relationship

between bundles $\{A, B\}$ and $\{C, D\}$, bidder 1 might choose to bid on $\{A, B\}$, which would lead to an inefficient allocation of the items: $\{A, B\}$ to bidder 1 for \$50. In fact, a simple trick makes it possible for model $(M1)$ to represent such cases. Consider an artificial item E with no value for any of the participants; Let bidder 1 submit the two bids $(\{A, B, E\}, \$50)$ and $(\{C, D, E\}, \$30)$, while bidder 2 submits the original bid $(\{B\}, \$30)$. Constraints 3 in $(M1)$ will ensure that $\{A, B, E\}$ and $\{C, D, E\}$ will not be allocated simultaneously to bidder 1. Nevertheless, this example shows that there is a need for richer and more expressive bidding environments enabling participants to describe succinctly and precisely their requirements.

To the best of our knowledge, the only successful attempt to define a complete bidding language for combinatorial bidding is due to Nisan (2000). The building blocks of the language are the atomic bids: pairs (S, p), where S is a subset of items and p is a price offer for S (this is basically the same definition we used in the previous section for the one-sided, indivisible case model). Then, OR bids and XOR bids are introduced. OR bids are collections of atomic bids with the implicit condition that any number of disjoint bids in the collection may be obtained for the sum of their respective price offers. XOR bids are also collections of atomic bids, but with the condition that at most one of the atomic bids in the collection may be obtained. The language is enriched by recursively combining OR and XOR bids in arbitrarily complex OR/XOR formulae. As a proof of validity, the author presents examples of valuation functions and shows that they can be correctly represented in the language.

We believe that Nisan's language, though an excellent starting point, needs to be developed further. The definition of XOR bids, or XOR operator, in particular, appears to be still somewhat restrictive and rigid. Consider the request "I am interested in the group of items $\{A, B, C, E, G\}$. I want to obtain two items and I am ready to pay \$100 for them. *Any two items in that group will be just fine*". Such requests are frequently encountered in numerous applications. Financial markets, in which traders sometimes bid on financial instruments in a sector of activity instead of specific assets, provide an illustration of such *indifferent bidding*. While this kind of requests could arguably be expressed as XOR bids through an exhaustive enumeration, this is certainly neither practical nor elegant. A better alternative would probably be the definition of new and more general selection operators "*Select K among N items*" (K-of-N) that could be applied to bundles as well as to single items in the expression of a bundle.

A second serious shortcoming of the language is that it is limited in scope to indivisible goods. It is legitimate to think that, as important applications of combinatorial auctions of divisible goods continue to emerge (financial markets, telecommunication capacity, etc.), a comprehensive and unified bidding framework, which would encompass both the divisible and the indivisible cases, would prove much more appropriate.

We are currently in the process of developing a unified bidding framework that should allow participants to express their bids as succinctly as possible, independently of any mathematical formulations. The corresponding mathematical programming formulations and solution approaches will then be developed as well as specific versions for particular applica-

tions. In the following, we limit ourselves to a methodological description of the broad lines of the bidding framework.

3.2 A glimpse into the bidding framework

The bidding vocabulary we propose is aimed at allowing bidders to formalize the structure of their bids and to specify particular requirements on their execution. These requirements are made of conditions and constraints under which participants will accept an allocation that may be synthesized in two fundamental principles: 1) a bid is a combination of bundles, and 2) a bundle itself is a combination of items. Yet, until now, one has generally considered a combined bid only as a collection of items, without specifying any structure to support them.

We propose a more flexible two-level representation of a combined bid (Abrache *et al.* 2001). At the lower, *inner* level, we define an atomic bid as a sell or buy request of a certain quantity of a single item, along with a price offer (our definition of an atomic bid is thus at a lower level then Nisan's). We then define *partial bids*, in which a collection of atomic bids is encapsulated along with *operators* that allow to express relationships between the atomic bids. A multi-level recursive definition may be given but, for the sake of simplicity, is not presented in this paper. Above this level, an *outer* level defines completely the structure of the bid. The outer level is mainly responsible for constructing the bid from several partial bids, imposing logical execution conditions, and specifying a price offer for the bid. It is important to note that, contrary to the inner level, the outer level is totally independant of the application, notably of the item divisibility condition. These concepts are illustrated below for the divisible case.

Definition 1 *Let G be a set of items. An atomic bid is a tuple $\delta = (r, q, p)$ where*

- *r: reference to an item, $r \in G$,*
- *q: quantity to trade,*
  - *$q > 0 \rightarrow$ buying item r*
  - *$q < 0 \rightarrow$ selling item r*
- *p: price offer.*

In the divisible case, q often designates the maximum quantity to trade, i.e., one may execute a fraction of an atomic bid only. Hence, we associate to each atomic bid δ a *proportion of execution* $x \in [0, 1]$ and the traded quantity of $\delta = (r, q, p)$ will be xq. Partial bids may then be defined:

Definition 2 *Let $\Delta_i = \{\delta_{ik}\}_{k=1,2,...,K_i}$ be a set of K_i atomic bids. Define a partial bid $\theta_i = [\Delta_i; \chi_i; p_i]$, where χ_i is an operator (to be defined later) applied to Δ_i, and p_i is a price offer for the partial bid.*

From a functional point of view, operators at the *inner level* may be classified in three categories.

1. *Combination* operators are aimed at imposing conditions on the traded proportions vector $x^i = \{x_{ik}\}_{k=1,2,...,K_i}$ associated to the atomic bids of θ_i. In general, a combination operator χ_i is defined parametrically to a subset $E \subseteq [0,1]^{K_i}$ and applying χ_i to Δ_i means that the condition $x^i \in E$ is imposed. For example, one of the combination operators we have developed is the **UANY** operator (Uniform value among the proportions), which is defined with respect to $E = \left\{ x \in [0,1]^{K_i} \,|x_k = x_{k+1}; k = 1, 2, ..., K_i - 1 \right\}$ and states that equal proportions of the atomic bids of θ_i have to be traded.

2. *Selection* operators are logical operators that allow to control the number of the atomic bids of a partial bid θ_i that have to be traded. The most general family of selection operators is

$$S_{k^l k^u}(\Delta_i) = \begin{cases} \textit{true}, & \text{if } k^l \leq |\Pi_i| \leq k^u, \\ \textit{false}, & \text{otherwise}, \end{cases}$$

   where $k^l \leq k^u \leq K_i$, and

$$\Pi_i = \{k \in \{1, 2, ..., K_i\} \,|x_{ik} > 0\}.$$

   Simply put, the selection operator $S_{k^l k^u}$ adds the condition that there should be no less than k^l and no more than k^u traded atomic bids (with $x_{ik} > 0$) among those specified by θ_i.

3. Hybrid operators that combine combination and selection operators.

Starting from the observation that a partial bid may be *executed* or not, the *outer level* is devoted to the specification of logical expressions involving two or more partial bids. For instance, given two partial orders θ_1 and θ_2 along with the basic logical operators AND, OR, and XOR, simple execution conditions can be defined:

- θ_1 AND θ_2 means that both partial bids must be executed;
- θ_1 OR θ_2 means that any one of the partial bids must be executed;
- θ_1 XOR θ_2 means that at most one of the partial bids must be executed.

Similarly to the inner level, it is also possible to define a more general selection operator. Let Ω be a set of I partial bids and consider the selection operator S_{N^l,N^u} defined as follows:

$$S_{N^l,N^u}(\Omega) = \begin{cases} \textit{true}, & \text{if } N^l \leq \mathcal{E}(\Omega) \leq N^u, \\ \textit{false}, & \text{otherwise}, \end{cases}$$

where $N^l \leq N^u \leq I$, and $\mathcal{E}(\Omega)$ is the number of the partial bids executed in Ω.

A bid may then be defined as

Definition 3 *Let $\Omega = \{\theta_i\}_{i=1,2,\ldots,I}$ be a set of I partial bids. A bid Θ can be defined such as: $\Theta = [\Omega; S_{N^l,N^u}; P]$, where S_{N^l,N^u} is a selection operator applied to Ω, and P is a price offer for the bid.*

The meaning and consistency of *prices* in the bidding vocabulary framework are important issues that need to be addressed. The reader has noticed that prices appear at all levels: bid, partial bid, and atomic bid. Which one of these values represents the actual price offer of the participant? Which values represent relative preferences only? For instance, when two items in a bundle are complementary or substitutable for a bidder, prices for the corresponding atomic bids are indeed not sufficient to characterize the price information the bidder may wish to submit. A price offer for the partial bundle that includes the two items is then required. In this case, atomic bid "prices" should most probably be interpreted as preference indicators rather than bid offers. Moreover, for certain applications, prices are not required at all levels. This important aspect of electronic markets, while strongly related to the negotiation mechanism (see Section 4), is still not totally understood yet. We plan to report new developments in the near future.

4 Tâtonnement processes

In a number of settings, knowing how to write bids and determine the winning allocation and prices is sufficient. Single-round, sealed-bid auctions are a case in point. Simply put, participants submit all their bids more or less simultaneously, and the auctioneer determines the winning bid by simply identifying the "best" bid with respect to pre-defined rules. The fact that bids are sealed implies that, in general, a bidder will have no information about other participants' behavior and, consequently, will derive its bidding strategy from incomplete and abstract (i.e., not related to the current auction) assessment of competition, as well as from its own valuation of the items on the market. Such a market is therefore inefficient (in an economic sense of the term) due, in particular, to a lack of information concerning the cost and utility functions of the market participants.

Assuming such information is available, one could build a model to determine optimal allocations and prices. To illustrate, consider an idealized multi-commodity, multi-lateral

market (Bourbeau *et al.* 2001). In this case, a market-maker asks the participants, sellers and buyers of products, to communicate all the relevant information about their production costs and demand functions, respectively. The market-maker also requires the transportation costs between sellers and buyers, as well as all the technological constraints related to the production and consumption of the products. The market-maker then solves a large non-linear market clearance model to identify an allocation and a set of equilibrium prices such that the total social efficiency is maximized.

Situations in which bidders hand over to market-makers complete and truthful information are very rare, however. The participants are generally *unwilling*, sometimes even *unable*, to disclose all the relevant information required by the auction mechanism to optimize the market. In this, they may be motivated by several considerations:

- Information confidentiality. Considering that participants are often self-interested agents, they are generally reluctant to disclose proprietary data.
- Uncertainty in the valuation of items. In some contexts, the value of items or bundles of items is not known with certainty to the participants and only estimates of the actual valuations can be communicated to the auctioneer (e.g., oil and gas lease auctions; see Oren and Williams 1975). In some other contexts, that information is imprecise (art auctions, for instance) and needs to be adjusted according to what competitors actually bid.
- Complexity of evaluating and communicating preferences. Especially when the number of items on the market is large and the bidding requirements of participants are complex, evaluation by participants of their own preferences can become a hard task. Moreover, a communication "bottleneck" exists (Nisan 2001) and implies that optimal outcomes in combinatorial auctions cannot be achieved, in the general case, with sub-exponential data communication.

Multi-round, tâtonnement processes alleviate some of these concerns since they require significantly less *a priori* information and allow participants to alter their selling or buying offers in light of the market information and their own assessment of the market. The idea of tâtonnement is the following. In each round, a market clearing mechanism - an optimization procedure in an optimized market - is used to produce a temporary set of allocations and prices. Then, the participants may alter their bids, or make new ones, according to their bidding strategies and pre-specified rules. The process continues until no new bids or bid updates are submitted (or other stopping criteria). Allocations and prices then become official. Bid changes from one round to the next are often governed by activity rules whose function is to give impetus to the market by prompting participants to be active in the market and reveal their real needs as early as possible. The design of these rules constitutes an important component of electronic auction mechanisms (e.g., Bourbeau *et al.* 2001 for the multi-lateral commodity market described earlier on).

Many "classical" auction mechanisms, such as the ascending English and the descending Dutch auctions, are tâtonnement processes. These are well known and understood. Tâtonnement processes in combinatorial auctions give rise to many complex problems, however. In the following, we focus on three particularly important aspects: the design of auction rules, the pricing of items or bundles of items, and the decision problems of participants.

Design of auction rules

A primary objective of the market-maker is to move the auction at a steady pace, ensuring that the participants progressively, but actively, commit themselves, and that the whole process eventually converges to allocations and prices close to what would have been obtained in the "ideal" case of an optimized, centralized market. Several rules must be set to achieve this goal:

1. Admissibility rules. These rules govern the way participants update their bids as the auction goes on. For the most part, they consist of constraints on the composition of bids (a bidder, for instance, may only bid on increasingly larger packages), or on price offers (bidder should bid incrementally on each bundle of items, for example).

2. Activity rules. In order to give impetus to the market, participants should express their real needs reasonably early in the auction. Hence, the mechanism should prevent, for instance, participants from simply observing the market, or making infinitesimal modifications to current bids, and waiting for the final stages of the auction to submit "jump" bids in an attempt to threw everybody else out of the auction.

3. Stopping rules. These rules specify criteria under which the process ends. Examples of these are predefined numbers of rounds, predefined auction times, and the absence of significant bidding activity. Note that a stopping rule can be made of the composition of several such stopping criteria.

Another focal aspect of an auction mechanism is its incentive compatibility properties. Ideally, a mechanism should be *strategy-proof*, i.e., it is a dominant strategy for participants to report their true valuations. In the single-round, sealed-bid case, the Generalized Vickrey Auction (GVA) is known to be an economically efficient, incentive compatible mechanism (see Krishna and Perry 1997, for instance). However, the GVA mechanism requires complete information about participants' valuation functions to be revealed to the auctioneer, and the design of alternatives for open multi-round environments is needed. Despite some important breakthroughs recently reported (e.g., Bikchandani *et al.* 2001, Parkes 2001), the problem is still open.

Pricing

The nature of the information disclosed to participants is an important issue in the design of tâtonnement processes. In single-item bidding auctions, the issue is easy to address: participants are informed of the winning bids and the new prices to beat on the single items. The issue gets significantly more complicated in combinatorial auctions, especially concerning the prices announced at the end of each round of the process.

Pricing in iterative combinatorial auctions is challenging essentially because, contrary to single-item bidding auctions, there is more than a one way to price items. The auctioneer could announce prices on single items and then compute bundle prices on the basis of the single-item prices (*linear pricing*). Or it could disclose bundle prices that are not necessarily supported by single-item prices (*non-linear pricing*). Finally, non-linear bundle prices could be *anonymous* (all the participants pay the same price if they win a given bundle), or not. But do "satisfactory" linear, bundle, anonymous or non-anonymous prices always exist?

Concerning linear pricing, Bikchandani and Mamer (1997) focus on an exchange economy with indivisibilities, which basically is the same setting as that of the one-sided, indivisible case, social efficiency maximization model ($M2$) of Section 2. The authors show that in this case, competitive equilibrium prices that support the optimal allocation (meaning that no other allocation gives a participant, at these prices, a higher surplus) exist if and only if the Pareto frontier of the economy coincides with the Pareto frontier of a transformed economy with divisible items only. In other terms, a necessary and sufficient condition for the existence of linear equilibrium prices is that the linear relaxation of model ($M2$) is integral. Under this condition, the dual of the linear relaxation of ($M2$)

$$(DLP2)\ \min \quad \sum_{j \in N} \tau_j + \sum_{i \in G} p_i \tag{17}$$

$$s.t. \quad \sum_{i \in S} p_i + \tau_j \geq v_j(S), \forall j \in N, \forall S \subseteq G, S \neq \emptyset \tag{18}$$

$$p_i \geq 0, \forall i \in G \tag{19}$$

$$\tau_j \geq 0, \forall j \in N \tag{20}$$

provides equilibrium prices $\{p_i\}_{i \in G}$ and participants' surpluses $\{\tau_j\}_{j \in N}$.

In general, however, model ($M2$) needs not be integral (see De Vries and Vohra 2001 for important instances in which it is). Consequently, alternative pricing schemes need to be developed. Bikchandani and Ostroy (1998) propose two alternative formulations that are equivalent to, but stronger than, model ($M2$). The first may provide non-linear, anonymous competitive equilibrium prices (that support the efficient allocation and at which no participant would obtain more utility from another allocation than the efficient one), but is still non-integral. The second alternative formulation is shown to be integral, and therefore always provides non-linear, non-anonymous competitive equilibrium prices. The mechanism iBundle (Parkes 2000) and its variants implement essentially these pricing schemes.

Other, "non-optimal", pricing methods exist. For instance, the experimental RAD mechanism of DeMartini *et al.* (1999) relies on an approximation of the linear pricing computed at the end of each round of the process. Wurman and Wellman (1999) prove the existence of anonymous bundle equilibrium prices and give a procedure to compute them. Their proof is constructive and proceeds in two steps. Once a provisional optimal allocation has been determined on the basis of bids submitted by the participants, prices for the assigned bundles are computed using a dual program, inspired from an early study by Leonard (1983), of an assignment problem. Then, prices of unassigned bundles are set such that no participant is distracted from the provisional optimal allocation. The equilibrium prices computed fail, however, to support the efficient allocation in general.

It is noteworthy that all previous issues have been studied on the basis of model ($M2$) or related formulations. It remains to determine how extensions to the more complex models that derive from the broad variety of applications supported by the bidding framework of Section 3 can be realized. A promising line of research, that we intend to explore, is the adaptation of Operations Research and mathematical programming methodologies through the exploration of the fundamental relationships between problem decomposition and tâtonnement processes (Dantzig 1963).

Participant decision problems

Participants need, of course, to construct initial bids and to modify them (both their composition and the associated price offers) as the tâtonnement process goes on. This is not the only problem faced by participants, however. To be able to decide on profitable bid strategies, participants have to analyze complex information disclosed by the market mechanism and combine it with their business processes: internal cost policies, current operations and activities, knowledge of the economic sector and competition, etc. Thus, there is need to develop optimization-based decision support tools - *(advisors)* - to help participants tackle those decisions.

To illustrate the types and role of advisors, consider an application to electronic freight marketplaces (Chang, Crainic, and Gendreau, 2001). Participants are shippers (production firms, freight forwarders, etc.) that need commodities (for simplicity, assume full load trailers or containers) moved between various locations, and motor carriers bidding for the loads. In designing their bidding strategies, carriers are faced with several questions: 1) on which loads to bid? 2) when to bid? and 3) at what prices? Decisions have to be consistent with the current and forecast fleet deployment and demand. It is also noteworthy that particular groups of loads may present a special interest for a given carrier when, for example, they may be blocked into a route performed by a single driver or they allow to bring home a driver and its empty vehicle. In this context, advisors are software agents that assist carriers in making "profitable" bidding decisions, by processing the information available in the market, and realize its integration into the dynamic planning of transportation operations.

A major difference between advisors and classical decision support systems is that while the latter have to interact "only" with the planning methods and data of the respective firm, the former have also to deal with the many forms of marketplaces encountered on the Internet. Consequently, other than the particular transportation sector in which they will evolve (truckload, less-than-truckload, container, a combination of the three, etc.), advisors may be classified according to their response to the following characteristics:

1. Market type. Advisors may be developed for *single* or *multiple* marketplaces. In the latter, carriers are interested in loads appearing on several different marketplaces. Indeed, while marketplaces are independent of each other, loads are often interdependent for carriers (e.g., to form an interesting route, loads have to be negociated on different marketplaces). In this case, advisors have the additional burden of coordinating the carriers' bidding activities in the different marketplaces. Advisors of this type have been proposed in the literature for very simple multi-market negotiations (Benyoucef *et al.* 2001), but no multi-market advisor is known to exist for more complex settings.

2. Auction type. Auctions can be *single* or *multi*-round, *continuous* or *periodic*, and may involve bidding on *independent single* loads, or on bundles (*combinatorial* bidding).

3. Integration with the planning process. Advisors can be *remotely coupled* to the (dynamic) planning of operations, or *tightly coupled* to it. In the first case, the advisor rely generally on predetermined lists of available vehicles and loads. On the other hand, tightly coupled advisors need to interact, at regular intervals with the (dynamic) fleet management model in order to evaluate loads to bid on. The length of those intervals depends on the response time of the model, as well as on the delays tolerated by the auction and the carrier tradeoff between profitability and risk of losing loads in the market.

In computer science terms, the advisor (or agent) *ménagerie* is even more diverse and complex. We may mention, for example, that while *planning* advisors, such as the ones described above, may be used to select loads on which to bid and determine the corresponding pricing data, *negotiators* are required to actually conduct the bidding. The complexity of the negotiation strategy, as well as its call to planning advisors during the auction, depend largely on the market characteristics and the time available for computations.

The development of dynamic advisors, for freight as for other types of markets, is thus a key design issue, especially critical for a successful deployment of combinatorial market designs.

5 Conclusion

Several complex issues arise in the design of combinatorial auctions of heterogeneous items. These issues go beyond the usual statements related to the complexity of solving the revenue

maximization problem. In this paper, we have illustrated the wide range of problems that may be encountered by presenting several formulations of combinatorial auctions in different settings. We have also discussed the need for expressive bidding vocabularies and emphasize the challenge of designing efficient tâtonnement mechanisms.

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